Laboratori Nazionali di Frascati

LNF-64/13 (1964)

M. Ademollo, R. Gatto, G. Preparata: TESTS FOR SPIN AND PARITY OF THE B MESON.

Estratto da: Phys. Rev. Letters 12, 462 (1964).

s in rather poor agreement with the direct result 31 ± 2.49) × 10^6 sec^{-1} of Alexander et al.; however, statistical errors are large. Moreover, the decay e for $K_2^0 \to \pi^+ + \pi^- + \gamma$ is unknown, and has been taken e zero in all published analyses. [It gives no back- und for the present result, Eq. (1).] We have not ished our analysis of the complete sample of K_2^0 d K_1^0) leptonic decays corresponding to our present nple of $\pi^+\pi^-\pi^0$ decays; we postpone a more detailed cussion of $\Gamma_2(L)$ until we have obtained its direct ermination from the complete sample. For a dis- ssion of the results of other relevant experiments, for example Luers et al., op. cit. (1964).

The decay rate $\Gamma_2(+-0)$ can also be obtained indirectly combining the K_2^0 lifetime τ_2 with the branching ratios $\Gamma_2(000)/\Gamma_2(\text{ch})$ and $\lambda = \Gamma_2(+-0)/\Gamma_2(\text{ch})$, under the as- nption that there are no additional unobserved neutral des, like $K_2^0 \to \gamma + \gamma$. R. H. Dalitz, Proceedings of Conference on the Fundamental Aspects of Weak eractions, Brookhaven National Laboratory, Upton, Y., September 1963 (unpublished), has combined the ilable data on τ_2, ν, and λ from eight different ex- iments, to obtain $\Gamma_2(+-0) = (1.93 \pm 0.35) \times 10^6\ sec^{-1}$. is result differs by 2.3 standard deviations from the ediction of Eq. (3). In the notation of Eq. (4), it rresponds to $\sqrt{2}\ \mathrm{Re}(A_{3/2}/A_{1/2}) = 0.12 \pm 0.06$.

The 3C production χ^2 distribution for the final 16 τ^0 ents is as follows: $\chi^2 = 0$ to 3.67, 8 events (we expect 20); 3.67 to 7.82, 5 events (we expect 4.00); 7.82 16.27, 3 events (we expect 0.78); >16.2, zero events expect 0.02). Thus the expected and observed χ^2 tributions are in excellent agreement. Since Λ pro- tion and decay occurs only about once in 30 pictures, re is only one chance in 900 of finding a 3-body K^0 ay with a possibly ambiguous origin. (We do not use gle-vee 3-body K^0 decays.)

N. P. Samios, Phys. Rev. 121, 275 (1961).

The 1C decay χ^2 distribution for the 16 final τ^0 events as follows: for $\chi^2 = 0$ to 1.07, 7 events (we expect 2); 1.07 to 3.84, 7 events (expect 4.00); 3.84 to 3, 2 events (expect 0.64); >6.63, zero events (expect 0.02).

[9]The decay $K_1^0 \to \pi^+ + \pi^- + \pi^0$ is forbidden for totally symmetric 3π states, and the contribution from non-symmetric states is expected to be small because of angular-momentum barrier effects. See, for instance, S. Treiman and S. Weinberg, Phys. Rev. 116, 239 (1959).

[10]The mean decay distance for K_2 is large compared to the bubble chamber. A small correction factor of 1.104 arises from the total attenuation by decay of the K_2's. The attenuation by interaction in the hydrogen is even less important and is neglected.

[11]This is our weighted average of the compilation by M. Chretien, V. K. Fischer, H. R. Crouch, Jr., R. E. Lanou, Jr., J. T. Massimo, A. M. Shapiro, J. P. Averell, A. E. Brenner, D. R. Firth, L. G. Hyman, M. E. Law, R. H. Milburn, E. E. Ronat, K. Strauch, J. C. Street, J. J. Szymanski, L. Guerriero, I. A. Pless, L. Rosenson, and G. A. Salandin, Phys Rev. 131, 2208 (1963), Table IV.

[12]This is our weighted average of the compilation by Frank S. Crawford, Jr., in Proceedings of the 1962 International Conference on High-Energy Physics at CERN (Cern Scientific Information Service, Geneva, Switzerland, 1962), p. 839.

[13]In the film analyzed at Wisconsin $t_0 = 0$ was used. At Berkeley t_0 corresponded to a cutoff at 0.8 cm. The time t_1 is the potential proper time corresponding to the decay fiducial volume. The production fiducial volume is slightly smaller than the decay fiducial volume, so that large values of $1/P$ are excluded.

[14]We impose no t_0 cutoff for τ^0 decays.

[15]In reference 3, the procedure was to use all of the acceptable Λ decays, irrespective of whether there is an acceptable K_1^0 decay, and sum over the calculated potential K^0 times. In that case one need not use the value of B. However, B is extremely well known, so that the two methods are equivalent. This was verified by comparing the methods in the film analyzed at Berkeley (75% of the total).

TESTS FOR SPIN AND PARITY OF THE B MESON

M. Ademollo, R. Gatto, and G. Preparata
Istituto di Fisica dell'Università di Firenze, Firenze, Italy
and Laboratori Nazionali del Comitato Nazionale per l'Energia Nucleare, Frascati, Roma, Italy
(Received 24 February 1964)

Experimental evidence has been recently re- rted of a π-ω resonance called B meson.[1-3] ferent assignments for its spin s and parity ave been proposed.[4-7] Methods for determin- s and P have been suggested[8,9] and prelimi- ry results seem to favor $s^P = 1^-$.[10] We pre- nt here alternative tests for determining the antum numbers and decay parameters of B. These tests are similar to those previously discussed for fermions.[11,12] We consider the cascade decay of B: $B \to \pi + \omega$ followed by $\omega \to \pi^+ + \pi^- + \pi^0$. We denote by $\rho_{\mu\mu'}^{(B)}$ (with $-s \leq \mu$, $\mu' \leq s$) the elements of the density matrix of B, in the B rest frame, normalized to unit trace. The density matrix of ω is then given, in the ω rest frame, by

$$I(\vec{v})\rho^{(\omega)} = \sum_{\mu\mu'} \rho_{\mu\mu'}^{(B)} \sum_{ll'} T_l T_{l'}^* \sum_{mm'} \sum_{\nu\nu'} (lm, 1\nu|s\mu)(l'm', 1\nu'|s\mu') Y_l^m(\vec{v}) Y_{l'}^{m'*}(\vec{v}) |\nu\rangle\langle\nu'|, \quad (1)$$

vhere $I(\vec{v})$ is the ω angular distribution. In Ξq. (1), $\vec{v}$ is the unit vector along the ω moment-ım; $|\nu\rangle$ is a spin eigenstate of ω $(-1 \leqslant \nu \leqslant 1)$; T_l is the reduced matrix element for B decay nto the lth final partial wave. If $P=(-1)^s$, $=l'=s$. If $P=(-1)^{s+1}$, l and l' can take on val-ıes $s \pm 1$. In terms of the irreducible tensor ›perators $T_\kappa{}^k$ we can write

$$\rho^{(\omega)} = \sum_{k=0}^{2} \sum_{\kappa=-k}^{k} Q_k{}^\kappa T_\kappa{}^{k\dagger}. \qquad (2)$$

The tensor operators $T_\kappa{}^k$ are normalized such hat $\mathrm{Tr}[T_\kappa{}^k T_{\kappa'}{}^{k'\dagger}] = \delta_{kk'}\delta_{\kappa\kappa'}$. The numerical :oefficients $Q_k{}^\kappa$ depend on the ω polarization: $Q_0 = 1/\sqrt{3}$; $\sqrt{2}\,Q_1{}^\nu$ are the spherical components of the ω polarization vector and cannot be measured from observation of $\omega \to 3\pi$ (they can be measured from $\omega \to \pi^0 + \gamma$); $Q_2{}^\sigma$ can be measured by averaging the function $Y_2{}^\sigma(\vec{n})$, where $\vec{n}$ is the normal to the decay plane of $\omega \to 3\pi$, over the distribution of $\vec{n}$[13]:

$$Q_2{}^\sigma = -(10\pi/3)^{1/2}\langle Y_2{}^\sigma(\vec{n})\rangle_{\vec{n}}. \qquad (3)$$

Comparing (2) with (1), by use of Racah's techniques, we find

$$I(\vec{v})Q_k{}^\kappa = \sum_{LM} a_k{}^\kappa(L,M) Y_L{}^M(\vec{v}); \qquad (4)$$

$$a_k{}^\kappa(L,M) = (-1)^k (4\pi)^{-1/2} (\hat{k}\hat{s}/\hat{L}) \sum_{f\varphi} \hat{f}^2 (f\varphi, k\kappa|LM) \sum_{ll'} (-1)^l T_l T_{l'}{}^* \hat{l}\hat{l}' (l0, l'0|L0) X \begin{Bmatrix} l & s & 1 \\ l' & s & 1 \\ L & f & k \end{Bmatrix} \sum_{\mu\mu'} \rho_{\mu\mu'} (s\mu', f\varphi|s\mu) \qquad (5)$$

vhere X is the Wigner 9-j coefficient and $\hat{a}$ $=(2a+1)^{1/2}$.[14,15] We note that $a_k{}^\kappa(L,M)$ vanishes or odd L. The coefficients $a_k{}^\kappa(L,M)$ satisfy the ·elation $a_k{}^{-\kappa}(L,-M) = (-1)^{M+\kappa} a_k{}^{\kappa *}(L,M)$. They :an be obtained directly from experiment as av-:rages on the ω angular distribution. Specifically,

$$(-1)^M a_k{}^\kappa(L,-M) = \langle Q_k{}^\kappa Y_L{}^M(\vec{v})\rangle_{\vec{v}}. \qquad (6)$$

The spin-parity tests are based on measure-nents of these averages. In practice one will neasure the averages as follows. To each event ›ne associates a B rest frame defined from the ›roduction reaction,[16] and a ω rest frame, direct-y obtained from it.[15] The averages (6), for $k=0$ ınd $k=2$, are approximated by[17]

$$\sqrt{3}\,\langle Q_0{}^0 Y_L{}^M\rangle \cong (1/N')\textstyle\sum_i{}' Y_L{}^M(\theta_i,\varphi_i), \qquad (7)$$

$$\sqrt{3}\,\langle Q_2{}^\kappa Y_L{}^M\rangle \cong -(10\pi)^{1/2}(1/N')\textstyle\sum_i{}' Y_2{}^\kappa(\alpha_i,\beta_i) \times Y_L{}^M(\theta_i,\varphi_i), \qquad (8)$$

vhere the subscript i refers to the ith event; the ırgument $(\theta_i,\varphi_i) \equiv \vec{v}_i$ is measured in the B rest system; $(\alpha_i,\beta_i) \equiv \vec{n}_i$ is measured in the ω rest system. The sum is over the N' events with dentical production kinematics (e.g., at same ›roduction angle in a two-body reaction); how-:ver, one can sum over all the production events vhen dealing with the linear tests that we propose. The tests are the following. We define the experimental test functions

$$A(Lk,f\varphi) = (12\pi)^{1/2} \sum_{M\kappa} \langle Q_k{}^\kappa Y_L{}^M\rangle (LM, k\kappa|f\varphi) \qquad (9)$$

[the normalization is such that $A(00,00)=1$]. From (6) and (5) it follows that

$$A(Lk,f\varphi) = \sqrt{3}\,\hat{k}\hat{s} R_s(f\varphi) \sum_{ll'} (-1)^l T_l T_{l'}{}^* \hat{l}\hat{l}' (l0, l'0|L0) \times X \begin{Bmatrix} l & s & 1 \\ l' & s & 1 \\ L & f & k \end{Bmatrix}, \qquad (10)$$

with

$$R_s(f\varphi) = \hat{s} \sum_{\mu\mu'} (-1)^{s+\mu} \rho_{\mu\mu'} (s\mu', s-\mu|f\varphi) \qquad (11)$$

[the normalization is $R_s(0,0)=1$]. Eq. (10) gives the test functions A as products of factors R depending on production and factors depending on decay. If $P=(-1)^s$, $l=l'=s$, and $|T_s|^2=1$, and we have

$$A(Lk,f\varphi) = (-1)^s \sqrt{3}\,\hat{k}\hat{s}^3 (s0, s0|L0) X \begin{Bmatrix} s & s & 1 \\ s & s & 1 \\ L & f & k \end{Bmatrix} R_s(f\varphi) \qquad (12)$$

so that the following conditions must be fulfilled: (i) $A(Lk,f\varphi)=0$ for $f+k=$ odd, due to the vanishing of the X coefficients; (ii) $A(Lk,f\varphi)/A(L'k',f\varphi)$ = known numbers depending on the assumed spin s. If, on the other hand, $P=(-1)^{s+1}$, l and l' asble I. The test functions for $P=(-1)^s$ [Eq. (12)].

s^P	
0^+	$A(00,00) = 1$
1^-	$A(00,00) = 1$
	$A(22,00) = 0.7071$
	$A(20,2\varphi) = 0.3162R_1(2\varphi)$
	$A(02,2\varphi) = -0.5000R_1(2\varphi)$
	$A(22,2\varphi) = -2.2136R_1(2\varphi)$
2^+	$A(00,00) = 1$
	$A(22,00) = 0.7071$
	$A(20,2\varphi) = -0.2673R_2(2\varphi)$
	$A(02,2\varphi) = -0.5916R_2(2\varphi)$
	$A(22,2\varphi) = -0.5051R_2(2\varphi)$
	$A(42,2\varphi) = -0.2711R_2(2\varphi)$
	$A(40,4\varphi) = -0.3564R_2(4\varphi)$
	$A(22,4\varphi) = 0.2020R_2(4\varphi)$
	$A(42,4\varphi) = 3.3128R_2(4\varphi)$

e values $s\pm1$ and the sum on the right-hand of (10) has the general form

$$\sum_{ll'}(\cdots)=a\alpha+ib\beta+c\gamma+d \qquad (13)$$

re a, b, c, and d are real numbers depending L, k, f, and s, and the decay parameters α, and γ are given by $\alpha=2\mathrm{Re}(T_{s-1}T_{s+1}^*)$, β m$(T_{s-1}T_{s+1}^*)$, and $\gamma=|T_{s-1}|^2-|T_{s+1}|^2$. ey satisfy $\alpha^2+\beta^2+\gamma^2=1$. The X coefficient in) gets multiplied by $(-1)^{f+k}$ by interchange of d l'. We thus have, for $f+k=$ odd, $a=c=d$ for $f+k=$ even, $b=0$. Taking the ratios $k,f\varphi)/A(L'k',f\varphi)$ we obtain a set of linear ations in the decay parameters. These ratios independent of the production process; there- e, for the experimental determination of the erages appearing in (9) as given by (7) and (8), can sum over all the events, considerably ducing the statistical error.[17] The conclusion that the set of equations obtained from (10) the different $Lk,f\varphi$ allows one to determine spin of B and [if $P=(-1)^{s+1}$] also its decay ameters.[18,19] For $s\leq2$ these equations are plicitly reported in Table I for $P=(-1)^s$ and Table II for $P=(-1)^{s+1}$. They can be numer- lly evaluated for larger spins directly from) using the available tables of the X coefficients.

Table II. The test functions for $P=(-1)^{s+1}$ [Eq. (10)].

s^P	
0^-	$A(00,00) = 1$
	$A(22,00) = -1.4142$
1^+	$A(00,00) = 1$
	$A(22,00) = -0.3536+0.3536\gamma+\alpha$
	$A(22,1\varphi) = -0.8660i\beta R_1(1\varphi)$
	$A(20,2\varphi) = (-0.1581+0.1581\gamma+0.4472\alpha)R_1(2\varphi)$
	$A(02,2\varphi) = (0.5500+0.4500\gamma)R_1(2\varphi)$
	$A(22,2\varphi) = (0.0598-0.0598\gamma+0.5916\alpha)R_1(2\varphi)$
	$A(42,2\varphi) = (0.3586-0.3586\gamma)R_1(2\varphi)$
2^-	$A(00,00) = 1$
	$A(22,00) = -0.3536+0.2121\gamma+1.0392\alpha$
	$A(22,1\varphi) = 0.8744i\beta R_2(1\varphi)$
	$A(20,2\varphi) = (0.0267-0.4009\gamma+0.1309\alpha)R_2(2\varphi)$
	$A(02,2\varphi) = (0.3803+0.2133\gamma)R_2(2\varphi)$
	$A(22,2\varphi) = (0.0196+0.1611\gamma+0.5677\alpha)R_2(2\varphi)$
	$A(42,2\varphi) = (0.2711-0.2711\gamma-0.2179\alpha)R_2(2\varphi)$
	$A(22,3\varphi) = 0.2498i\beta R_2(3\varphi)$
	$A(42,3\varphi) = -0.3938i\beta R_2(3\varphi)$
	$A(40,4\varphi) = (-0.0891+0.0891\gamma-0.4364\alpha)R_2(4\varphi)$
	$A(22,4\varphi) = (-0.4108-0.4377\gamma+0.0495\alpha)R_2(4\varphi)$
	$A(42,4\varphi) = (0.3333-0.3333\gamma-0.3245\alpha)R_2(4\varphi)$
	$A(62,4\varphi) = (-0.2205+0.2205\gamma)R_2(4\varphi)$

M. Abolins, Richard L. Lander, Werner A. W. hlhop, Nguyen-huu Xuong, and Philip M. Yager, ys. Rev. Letters 11, 381 (1963).

[2]S. U. Chung et al., Proceedings of the 1963 Siena Conference on Elementary Particles, Siena, Italy, 1963 (unpublished).

[3]G. Goldhaber et al. (to be published).

[4]W. R. Frazer, S. H. Patil, and N. Xuong, Phys. Rev. Letters 12, 178 (1964).

[5]R. F. Peierls, Phys. Rev. Letters 12, 50 (1964).

[6]C. Goebel (to be published).

[7]E. Abers, Phys. Rev. Letters 12, 55 (1964).

[8]A. C. Zemach (to be published).

[9]F. R. Halpern, Phys. Rev. Letters 12, 252 (1964).

[10]D. Duane Carmony, Richard L. Lander, Carl Rindfleisch, Nguyen-huu Xuong, and P. Yager, Phys. Rev. Letters 12, 254 (1964).

[11]M. Ademollo and R. Gatto, Nuovo Cimento 30, 429 (1963).

[12]M. Ademollo and R. Gatto, Phys. Rev. 133, B531 (1964).

[13]An alternative expression is: $Q_2^{\sigma}=2(10\pi/3)^{1/2}\langle Y_2^{\sigma}(\vec u)\rangle_{\vec u}$ where $\vec u$ is the unit vector along the momentum of any one of the final pions.

[14]The angular distribution is obtained from (4) and (5) for $k=\kappa=0$:

$$\sqrt{3}a_0^{\,0}(L,M)=(-1)^{S+1+L/2}(4\pi)^{-1/2}(1/\hat s)$$
$$\times\sum_{ll'}(-1)^{(l'-l)/2}T_lT_{l'}^*Z(lsl's;1L)$$
$$\times\sum_{\mu\mu'}\rho_{\mu\mu'}^{(B)}(s\mu',f\varphi|s\mu),$$

where we have used the Biedenharn Z coefficients.

[15]Although the formalism is nonrelativistic, the results are relativistically correct if some care is exerted. The system in which the Q_k^K are measured must be that ω rest system which is obtained from the B rest system by a pure timelike Lorentz transformation. See, e.g., H. P. Stapp, Phys. Rev. 103, 425 (1956), for a detailed discussion.

[16]For example, if B is produced in $\pi+N\rightarrow N+B$, one first takes the reaction c.m. system with x along the incident nucleon and z normal to the production plane; one then goes to the B rest frame by a pure timelike Lorentz transformation.

[17]For the discussion of the finite samples see section 4 of reference 11.

[18]Additional tests derive from the conditions $\rho_{\mu\mu}^{(B)}\geq 0$ and $\sum_\mu \rho_{\mu\mu}^{(B)}=1$. From (11) one has $\rho_{\mu\mu'}^{(B)}=(-1)^{s+\mu}\times(2s+1)^{-1/2}\sum_{f\varphi}R_s(f\varphi)(s\mu', s-\mu|f\varphi)$. In some case [viz.: $k=0$ or $P=(-1)^s$] $R_s(f\varphi)$ for odd f cannot be determined and one can only derive $\rho_{\mu\mu'}^{(B)}+(-1)^{\mu-\mu'}\times\rho_{-\mu'-\mu}^{(B)}$. One obtains

$$(-1)^{s+\mu}\sum_f R_s(f\varphi)(s\mu, s-\mu|f0)\geq 0$$

and

$$|R_s(f0)|\leq(2f+1)^{1/2}\max|(s\mu, f0|s\mu)|$$

where max is meant with respect to μ. The R_s are experimentally found from Eq. (10).

[19]Additional tests can be derived if B is produced in the two-body reaction $\pi+N\rightarrow N+B$ with unpolarized nucleons. As a consequence of parity conservation $\rho^{(B)}$ has nonzero elements only for $\mu-\mu'$ even. We write $\rho^{(B)}=\rho_e+\rho_0$, where ρ_e has only the elements of $\rho^{(B)}$ with $s-\mu$ and $s-\mu'$ both even and ρ_0 those with $s-\mu$ and $s-\mu'$ both odd. We can show that ρ_e and ρ_0 must have rank 2. For example, for $s=2$ one must have $\mathrm{Det}\|\rho_e\|=0$.

MODEL FOR THE $\pi\omega$ RESONANCE AT 1220 MeV *

T. K. Kuo
Brookhaven National Laboratory, Upton, New York
(Received 27 January 1964)

Since the recent discovery of the B particle,[1] a number of authors[2] have discussed its possible quantum numbers. In particular, a dynamical scheme, in analogy with πN scattering, was suggested by Peierls[3] and by Abers.[4] In this approach the $\pi\omega$ scattering is considered to be elastic near threshold, and the dominant "force" for the scattering is attributed to the ρ meson. However, whereas Peierls, who used the large mass limit and therefore ignored s-wave scattering completely, obtained a possible resonance in the 2^- state, Abers, employing a relativistic calculation, suggested a 1^+ resonance. We have also performed a relativistic calculation along the same lines. The result resembles the corresponding πN scattering in that the "Born terms" are large and of "long range" for p-wave scattering, and are large but of "short range" in s-wave scattering. Provided that one believes that at low energies short-range scatterings are relatively structureless, then one can conclude that here, as in πN scattering, only p-wave scattering is important and a resonance in 2^- state is the counterpart of the 3-3 resonance.

We begin with the two diagrams, Figs. 1(a) and 1(b). The $\omega\rho\pi$ vertex is

$$f\epsilon_{\mu\nu\lambda\sigma}\epsilon_{(\omega)}^{\mu}k_{(\omega)}^{\nu}\epsilon_{(\rho)}^{\lambda}q_{(\rho)}^{\sigma}, \qquad (1)$$

in an obvious notation. The units are chosen so that $\hbar=c=m_\pi=1$. Then $f^2/4\pi=0.45$,[2] for an ω width of 9 MeV and a ρ width of 100 MeV. The various angular momentum states can be projected out for Figs. 1(a) and 1(b) by the use of helicity amplitudes in the barycentric system. The helicity of the ω meson is labelled by +, -, and 0. Parity conservation and time-reversal invariance reduce the possible nine helicity amplitudes to four independent ones. (A corresponding statement holds for angular momenta eigenamplitudes with $l=J$, $J\pm1$.)

It is found that, for Fig. 1(a),

$$f_{00}=f_{+0}=0,$$

$$f_{++}=-\left(\frac{f^2}{4\pi}\right)\frac{k^2s}{2W}\frac{d_{1,1}^{1}(\theta)}{(s-m^2)},$$

$$f_{+-}=\frac{f^2}{4\pi}\frac{k^2s}{2W}\frac{d_{1,-1}^{1}(\theta)}{(s-m^2)}; \qquad (2)$$

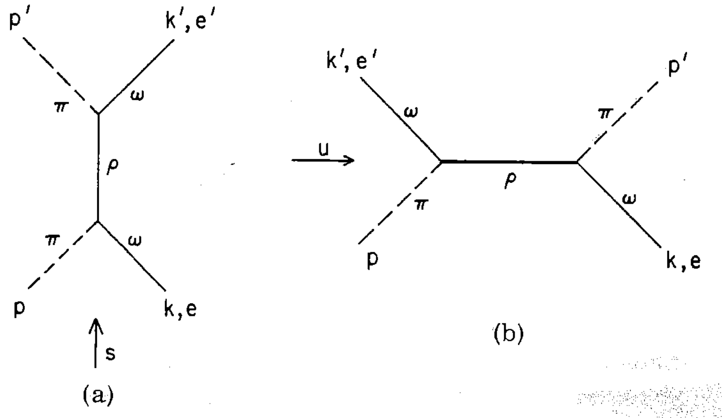

FIG. 1. (a) Direct and (b) exchanged ρ contributions.